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Engineering Notes

Flutter Analysis: Using Piecewise Quadratic Interpolation with Mode Tracking and Wind-Tunnel Tests

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Nomenclature

b	=	reference length of half-chord
b_j	=	breakpoint of the jth segment
c_0, c_1, c_2, c_3, c_4	=	coefficients of interpolation polynomial
$\mathbf{D}$	=	generalized damping matrix
$f(V)$	=	cubic spline function of flow velocity
f_i	=	frequency corresponding to the flow velocity at the ith step
g	=	added structural damping for k method
Im()	=	imaginary part of complex number
j	=	sequence number of segment
$\mathbf{K}$	=	generalized stiffness matrix
k	=	reduced frequency
k_j	=	the jth reduced frequency in which the generalized aerodynamic matrix is computed
M	=	Mach number
$\mathbf{M}$	=	generalized mass matrix
n	=	number of reduced frequencies for computing aerodynamic influence matrices
n_a	=	number of frequency data in a set
p	=	Laplace variable
$\mathbf{Q}_s(p)$	=	rational function approximation for generalized aerodynamic matrix
$\mathbf{Q}_j^0, \mathbf{Q}_j^1, \mathbf{Q}_j^2$	=	complex-valued coefficient matrices of the jth segment
$\mathbf{q}$	=	vector of generalized coordinates
V	=	flow velocity
V_i	=	flow velocity at the ith step
ρ	=	flow density

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I. Introduction

MOST methods for flutter analysis require the interpolation of aerodynamic forces in frequency domain and the iteration procedure for solving nonlinear eigenvalue problems [1–3]. Goodman [4] proposed an excellent approach to converting the problem of flutter analysis to a piecewise quadratic eigenvalue problem. This approach, however, does not impose any constraints on the derivatives of interpolation functions so that the eigenvalues may be discontinuous on the segment boundaries, exhibiting gaps or overlaps. Eller [5] proposed an improved scheme by dividing the reduced frequency range into $n-2$ segments with $n-1$ breakpoints, keeping the continuity of the function derivatives on the segment boundaries. Nevertheless, the previous studies on the piecewise quadratic interpolation (PQI) method mainly focused on the interpolation improvements. No studies have dealt with the mode tracking jumps, which will appear in v-g and v-f diagrams if the reduced frequencies are not properly chosen. In addition, no wind-tunnel tests have verified the efficacy of the PQI method in flutter analysis.

The objective of this Note is to make a study on the flutter analysis of PQI. The rest of the Note is organized as following. In Secs. II and III, a shape-preserving cubic spline for mode tracking is proposed and then numerically illustrated through the aeroelastic analysis of a 15 deg sweptback wing at the Mach number of $M=0.45$. In Sec. IV, the numerical simulations and corresponding wind-tunnel tests of a flexible wing model are presented for further verification of the proposed method. Finally, a few concluding remarks are made in Sec. V.

II. Mode Tracking Scheme

The dynamic equation in flutter analysis can be transformed to the following equation by using the Laplace transform:

$$\left[\frac{V^2}{b^2}\mathbf{M}p^2+\frac{V}{b}\mathbf{D}p+\mathbf{K}-\frac{1}{2}\rho V^2\mathbf{Q}_s(p)\right]\mathbf{q}=\mathbf{0} \tag{1}$$

Once the piecewise quadratic function of $\boldsymbol{Q}_s(p)$ is available, Eq. (1) can be recast as

$$\left[\left(\frac{V^2}{b^2}\mathbf{M}-\frac{1}{2}\rho V^2\mathbf{Q}_j^2\right)p^2+\left(\frac{V}{b}\mathbf{D}-\frac{1}{2}\rho V^2\mathbf{Q}_j^1\right)p+\mathbf{K}-\frac{1}{2}\rho V^2\mathbf{Q}_j^0\right]\mathbf{q}=\mathbf{0} \tag{2}$$

This is a standard quadratic eigenvalue problem for each segment. However, It is necessary to track the eigenvalues in the PQI method. The PQI method in Eller's scheme is C^1 continuous on the stability boundary, offering inherent facility to reduce the possible discontinuities at large damping values to arbitrarily small levels [5]. Therefore, neither mode tracking nor blending of roots is required on the boundaries. However, if the reduced frequencies are not chosen properly and the number of segments is not large enough, a sudden jump may occur in mode tracking. As shown in Fig. 1, several tracking jumps between modes 2 and 3 appear in the solutions of the PQI method for a 15 deg sweptback wing at the Mach number of $M=0.45$. That is, the computed dampings and frequencies of modes 2 and 3 exchange each other all of a sudden. The main reason for such a phenomenon is that two (or even more) roots, which yield the condition of $\mathrm{Im}(p)\in[b_j,b_{j+1}]$, are kept in segment j but not sorted out. If these roots are not tracked properly, the frequency and damping values of one mode will be incorrectly used.

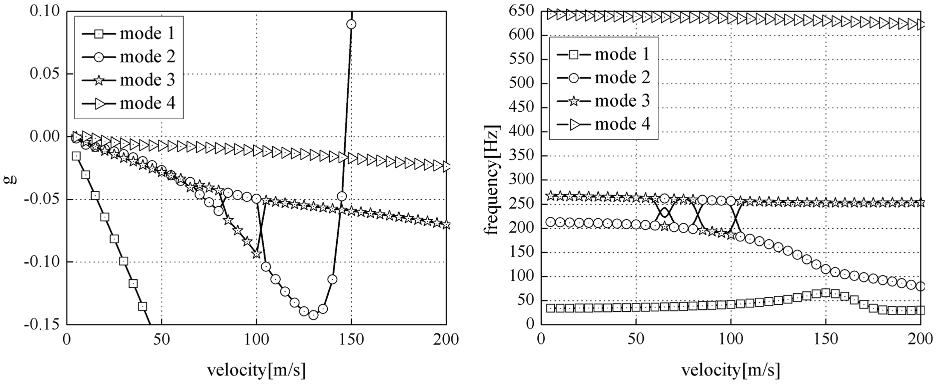

Fig. 1 The solutions of the PQI method without mode tracking for a 15 deg sweptback wing at $M = 0.45$.

The issue of eigenvalue tracking can be handled by means of shape-preserving cubic spline extrapolation [6], the advantage of which is effective and simple for mode tracking. The other advantage of the scheme is that it can be executed by using a numerical library, such as the International Mathematical and Statistical Library (IMSL) C Numerical Library. The cubic spline is a smooth polynomial of the fourth order. The IMSL Numerical Library provides two cubic splines. The first spline enables one to specify various endpoint conditions, such as the first or the second derivatives at the right and left endpoints, and the other spline preserves the shape of curve-matching interpolation data. Thus, it is better for mode tracking to adopt the second spline in flutter analysis.

Given a set of frequency data corresponding to different flow velocities,

$$f_i = f(V_i); \qquad 1 \leq i \leq n_a \tag{3}$$

the shape-preserving interpolation function is defined as the following piecewise polynomial of the fourth order:

$$f(V) = c_0 + c_1(V - V_i) + c_2(V - V_i)^2 + c_3(V - V_i)^3 + c_4(V - V_i)^4; \qquad V_i \leq V \leq V_{i+1} \tag{4}$$

To determine the coefficients of the interpolation polynomial for each interval $[V_i, V_{i+1}]$, the function values f_i and f_{i+1}, and the first derivatives f'_i and f'_{i+1} are needed. The first derivative of the interpolation function at V_i can be estimated from the frequency data at V_i and its two neighbors [6]. The shape-preserving interpolation function can be solved by using the IMSL function *imsl_d_cub_spline_interp_shape*.

Figure 2 depicts the block diagram of the PQI method with mode tracking for flutter analysis. First, the complex-valued coefficient matrices are obtained by using PQI, which is simply computed by solving a set of complex-valued linear algebra equations. Afterward, the frequencies corresponding to the first several flow velocities are saved as sample data for the cubic spline interpolation, and then the predicted frequency of each mode at the next step of flow velocity is extrapolated by using the IMSL cubic spline shape-preserving function *imsl_d_cub_spline_interp_shape*. According to the minimum difference between the computed frequency and the predicted frequency, the frequencies of all modes can be sorted out, as shown in Fig. 3.

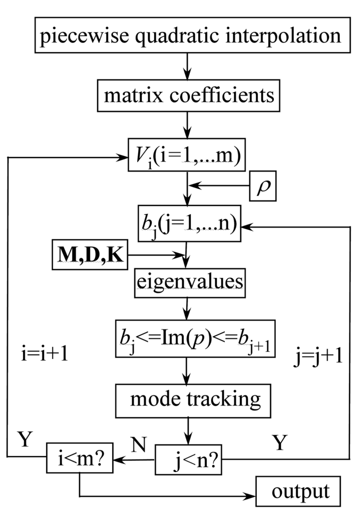

Fig. 2 The block diagram of the PQI method with mode tracking.

III. Illustrative Example

This section presents how to apply the PQI method to the aeroelastic analysis of a 15 deg sweptback wing at a flow velocity of $M = 0.45$, a standard example of flutter analysis in MSC Nastran. The modal parameters, such as generalized mass, natural frequencies, and mode shapes, of the wing were obtained from the MSC Nastran result file through direct data abstraction. The aerodynamic

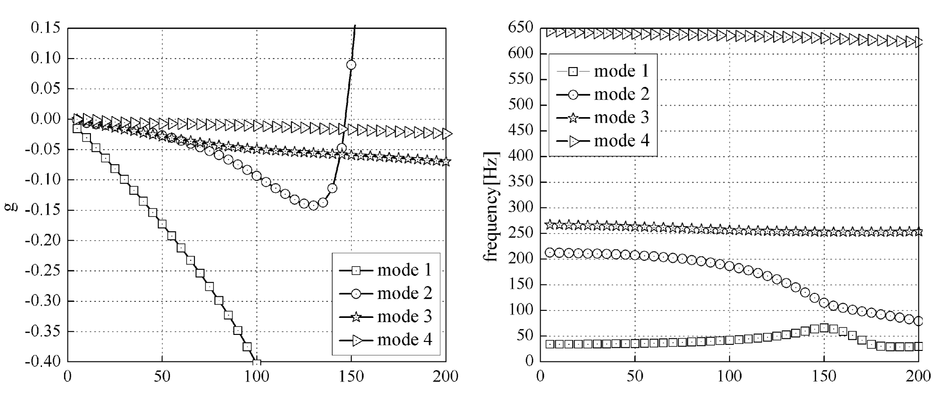

Fig. 3 The solutions of the PQI method with mode tracking for a 15 deg sweptback wing at $M = 0.45$.

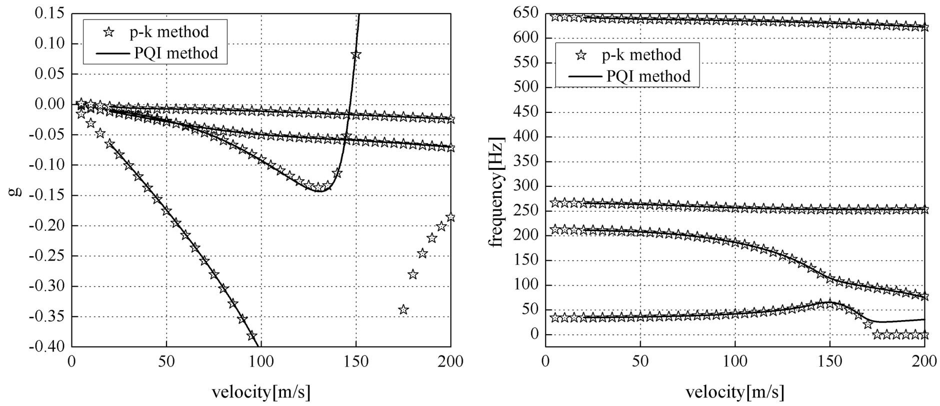

Fig. 4 The *v*-*g* and *v*-*f* diagrams computed by using different methods for a 15 deg sweptback wing at $M = 0.45$.

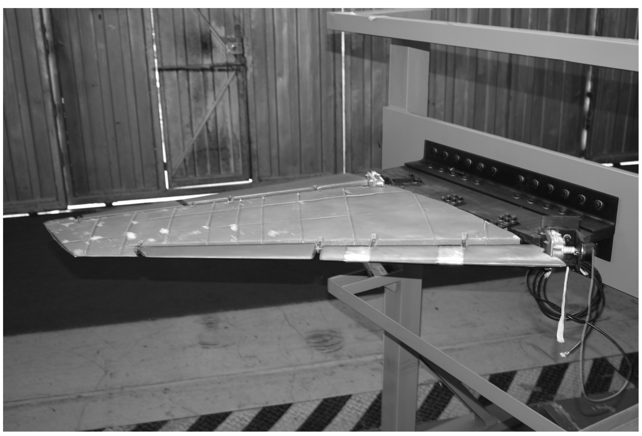

Fig. 5 The wind-tunnel model of a wing with four control surfaces.

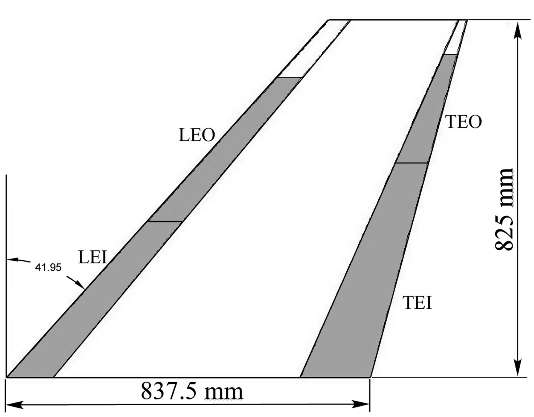

Fig. 6 Configuration of four control surfaces of wing model (LEI denotes leading-edge inboard, and TEI denotes trailing-edge inboard).

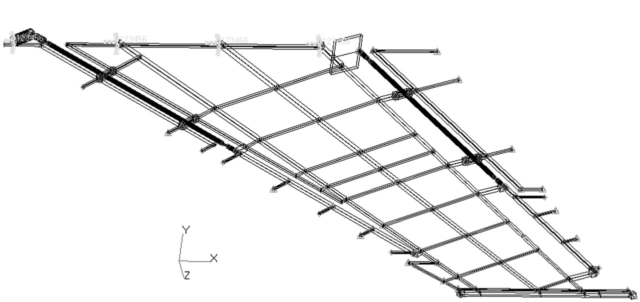

Fig. 7 The finite-element meshes of the wing model.

influence coefficient matrix [7], the surface spline interpolation [8], and the flutter analysis were computed in a flutter analysis/aeroservoelastic analysis package written in C++ in Microsoft Visual Studio 2008. The linear algebra computation was completed by using the IT++ software package. In this case study, two methods for flutter analysis were used. The first is the *p*-*k* method and the second is the PQI method. Figure 4 shows the v-g diagram of damping ratios, with respect to flow velocity, and the v-f diagram of flutter frequency, with respect to flow velocity, both computed by using two methods. The flutter solutions of the PQI method get a good agreement with those of the *p*-*k* method as expected, until the flow velocity reaches 175 m/s. When the flow velocity approaches 175 m/s, there is a tracking jump in the solution of the *p*-*k* method for mode 1. However, the PQI method tracks all eigenvalues very well. It is noted that the eigenvalues with $|g| > k$ should be considered to be dubious, regardless of the method of solution [5].

IV. Numerical and Experimental Studies for a Wing Model

This section presents the detailed aeroelastic analysis, based on the PQI method, for a wind-tunnel model of a flexible wing designed for weight savings by the additional use of active control. As shown in Fig. 5, the model is a low-aspect ratio wing with a span of 0.825 m, made of an aluminum framework covered by plastic foam for aerodynamic configuration. The wing has two leading-edge and two trailing-edge control surfaces, as shown in Fig. 6. In this study, however, only the leading-edge outboard (LEO) and the trailing-edge outboard (TEO) control surfaces are connected to the wing by hinge-line-mounted, rotary ultrasonic actuators. Figure 7 presents a three-dimensional configuration of the finite-element model of the wing frame. To adapt the velocity slope of the wind tunnel, the wing model was supported elastically.

A. Modal Analysis

As the first step of the study, the modal analysis of the wing model was made via the conventional finite-element method and the ground vibration test. In the ground vibration test, the plastic foam covering the wing model makes it difficult to measure the dynamic response of the wing model via accelerometers fixed on the foam or excite the wing model via a hammer impacting on the foam. Hence, a laser interferometer was used to measure the dynamic responses, subject

Table 1 Comparison of computed and measured natural frequencies of wing model

Computed frequency, Hz	Measured frequency, Hz	Error, %	Description of mode shapes
3.1753	3.498	10.163	First bending mode
8.2302	9.891	20.179	First torsional mode
13.891	14.774	6.357	Second bending mode
18.265	21.772	19.201	Second torsional mode

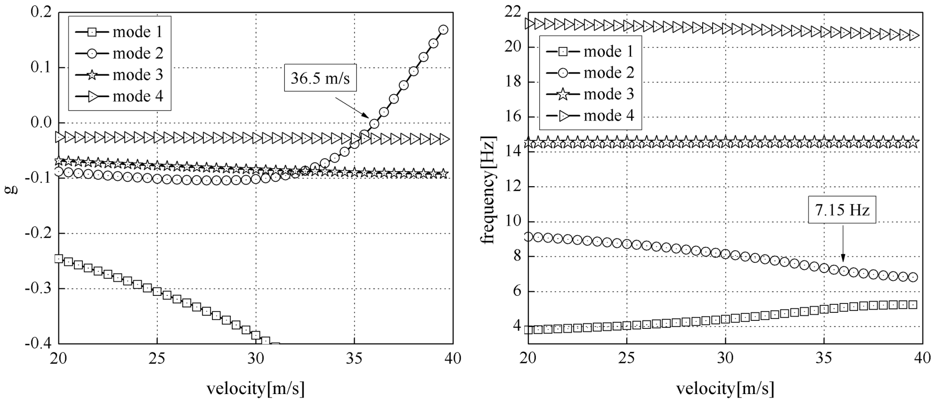

Fig. 8 The *v*-*g* and *v*-*f* diagrams of the wing model at $M = 0.05$.

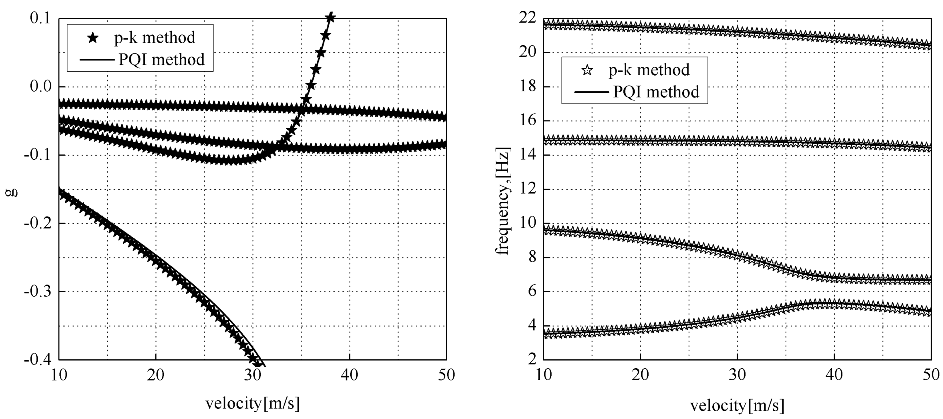

Fig. 9 The *v*-*g* and *v*-*f* diagrams computed by using different methods for the wing model at $M = 0.05$.

to a hammer impact, at different locations on the wing model. Two ground vibration tests were performed. In the first test, the excitation of the hammer was kept to the spring piece of the trailing edge of the wing root and the laser interferometer was put at 49 different locations. The first four natural frequencies, damping ratios, and mode shapes were identified through the measured data. However, the first bending mode shape was not good enough because of an improper excitation location. In the second test, the excitation of the hammer was moved to the trailing edge of the wing tip. The above modal parameters were identified again from the measured data. Compared with the results in the first test, the first bending mode shape became much better, while the remaining three mode shapes were poorer. Therefore, the first bending mode shape in the second test and the remaining three mode shapes in the first test were collected for later use. Table 1 shows that the first four natural frequencies measured are higher than the computed natural frequencies. The main reason come from the plastic foam, which was neglected in modeling the wing by using MSC Nastran. The first four damping ratios measured are 3.007, 1.81, 1.163, and 1.143%, respectively.

B. Aeroelastic Analysis

The PQI method with mode tracking was used to compute the flutter of above wing model. The first four natural frequencies and corresponding mode shapes used for flutter analysis were corrected through measured values from ground vibration tests, while the damping ratios of the first four modes were appended to the dynamic equation of the aeroelastic system. Figure 8 shows that the PQI method with the shape-preserving cubic spline mode tracking can correctly predict the flutter velocity and frequency. In Fig. 8, the computed flutter velocity and frequency of the wing model are

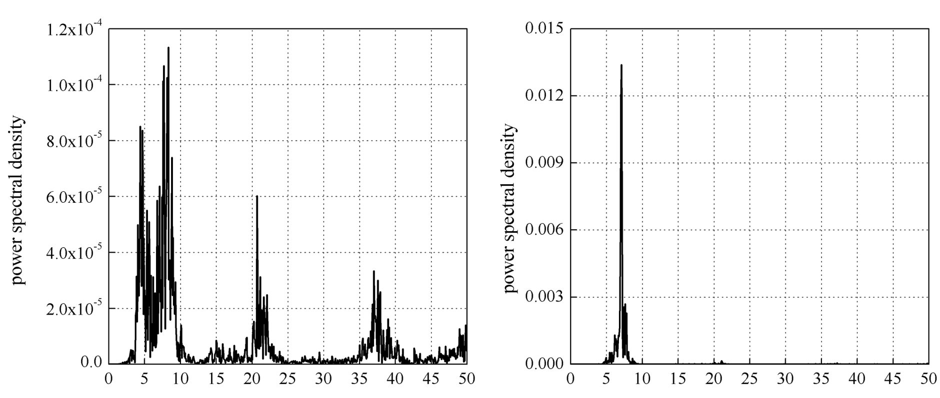

Fig. 10 The power spectral density of the wing model response measured at flow velocities of $V = 32$ m/s and $V = 38.5$ m/s.

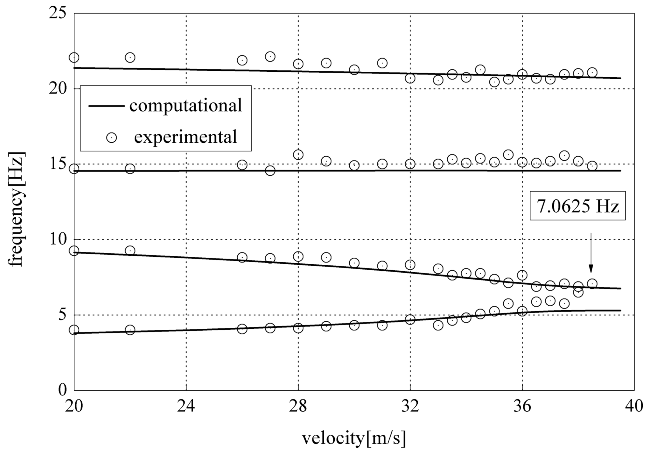

Fig. 11 The computational and experimental *v-f* diagrams of the wing model.

36.5 m/s and 7.15 Hz, respectively. In addition, the p-k method was also used to compute the flutter of the wing model. Figure 9 shows the v-g and v-f diagrams computed by using the p-k method and the proposed method, respectively. As shown in Fig. 9, the flutter frequencies predicted by the two methods agree well with each other. However, the damping ratios computed by using the p-k method are larger than those obtained from the PQI method.

C. Wind-Tunnel Test

The wind-tunnel tests were conducted at the Nanjing University of Aeronautics and Astronautics NUAA-2 low-speed wind tunnel. The wing model was supported by a steel stand within the test section of the wind tunnel, and the response signals of the wing model were sensed by two accelerometers fixed at the leading edge and trailing edge of the wing tip, respectively. As shown in Fig. 10, the spectrum of the flow-induced dynamic response of the wing model became single-mode dominated when the flow velocity increased to 38.5 m/s. In this case, the natural frequencies of the first two modes met each other, indicating the coupled, bending torsional flutter with a very slight increase in flow velocity. Figure 11 shows that the measured flutter frequency of the wing model is 7.0625 Hz, very close to 7.15 Hz (the computed flutter frequency of the wing model). The computational errors of flutter frequency and velocity are 1.24 and 5.2%, respectively. They may come from the reduced aerodynamic pressure, owing to many small backlashes in the wing model.

V. Conclusions

The mode tracking may undergo sudden jumps in the flutter solutions of the PQI method. A simplified shape-preserving cubic spline scheme is added to this method so as to track all modes of an aeroelastic system properly. The proposed mode tracking scheme can be easily executed by using a numerical library, such as the IMSL C Numerical Library. Numerical results of the flutter analysis for a 15 deg sweptback wing at a flow velocity of $M = 0.45$, a standard example of MSC Nastran, show that the shape-preserving cubic spline extrapolation, used for mode tracking, could track all modes of the wing efficiently. The wind-tunnel tests performed by the authors for a flexible wing model verify the efficacy of the revised PQI method. The experimental results show that the proposed method provides accurate predictions of flutter velocity and frequency of the wing model.

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